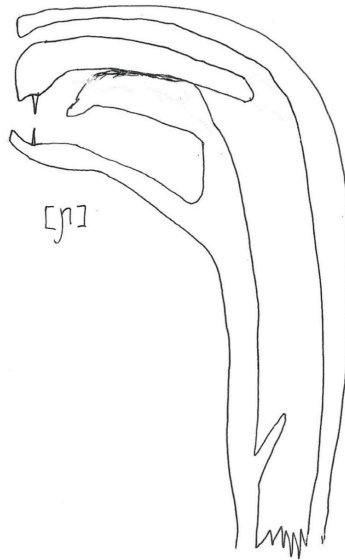
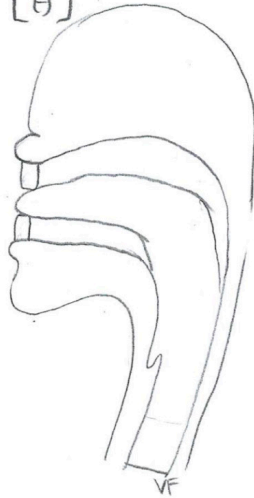


Harmonics

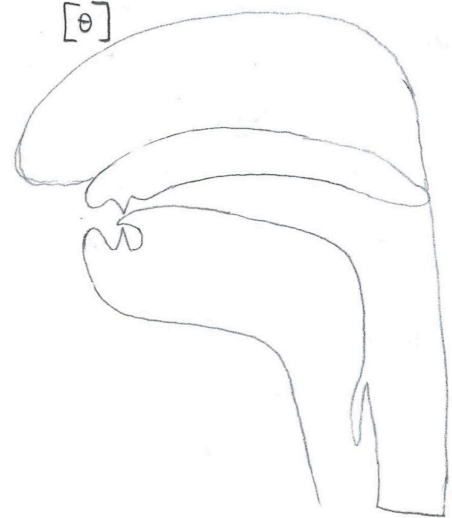
A Mid-Sagittal Interlude



[ə]



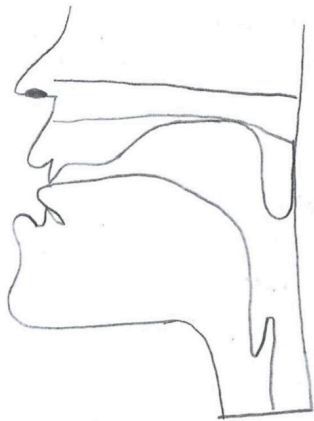
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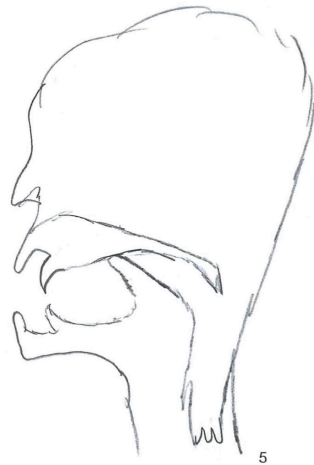
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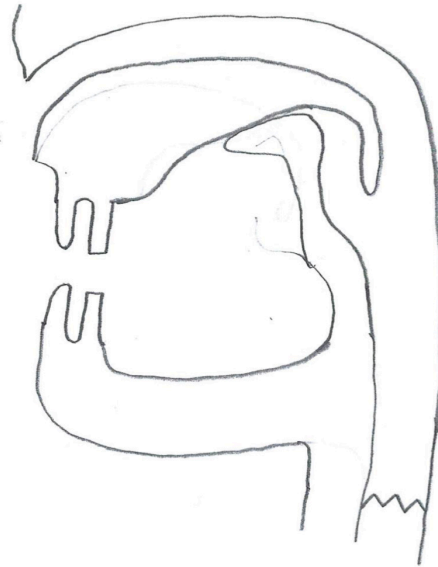
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[2]



drawing is hard.



moral of
story: tongues
are f***ing
weird

Where Are We Going?

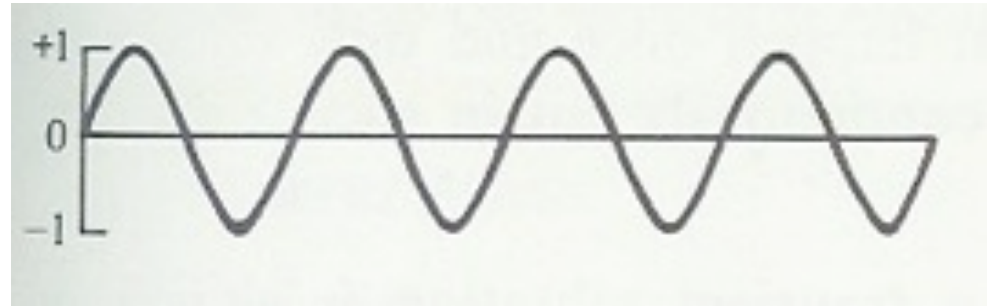
- For the next week or so: more acoustics
 - Start Harmonics today, Resonance next time
- Just so you know:
 - It's going to get worse before it gets better
- Note: A supplementary reading on today's lecture has been posted to the course web page.
- What have we learned?
 - How to calculate the frequency of a periodic wave
 - How to calculate the amplitude, RMS amplitude, and intensity of a complex wave
- Today: we'll start putting frequency and intensity together

Complex Waves

- When more than one sinewave gets combined, they form a **complex wave**.
- At any given time, each wave will have some amplitude value.
 - $A_1(t_1)$:= Amplitude value of sinewave 1 at time 1
 - $A_2(t_1)$:= Amplitude value of sinewave 2 at time 1
- The amplitude value of the complex wave is the sum of these values.
 - $A_c(t_1) = A_1(t_1) + A_2(t_1)$

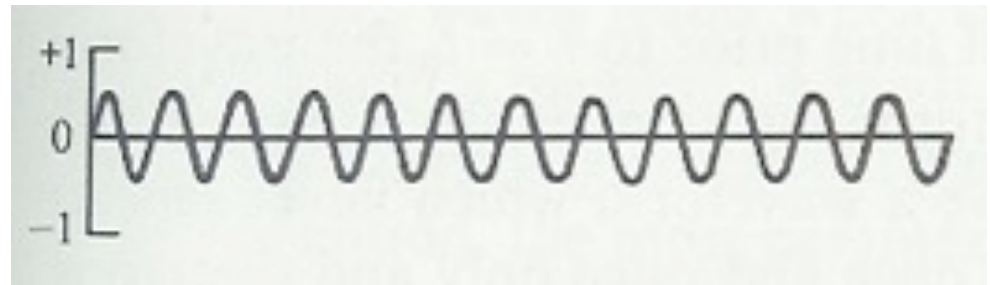
Complex Wave Example

- Take waveform 1:
 - high amplitude
 - low frequency



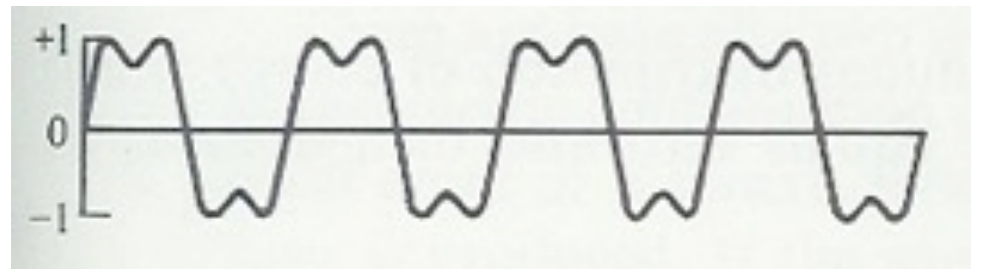
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- Add waveform 2:
 - low amplitude
 - high frequency



=

- The sum is this complex waveform:



Greatest Common Denominator

- Combining sinewaves results in a **complex periodic wave**.
- This complex wave has a frequency which is the **greatest common denominator** of the frequencies of the component waves.
- Greatest common denominator = biggest number by which you can divide both frequencies and still come up with a whole number (integer).
- Example:
 - Component Wave 1: 300 Hz
 - Component Wave 2: 500 Hz
 - Fundamental Frequency: 100 Hz

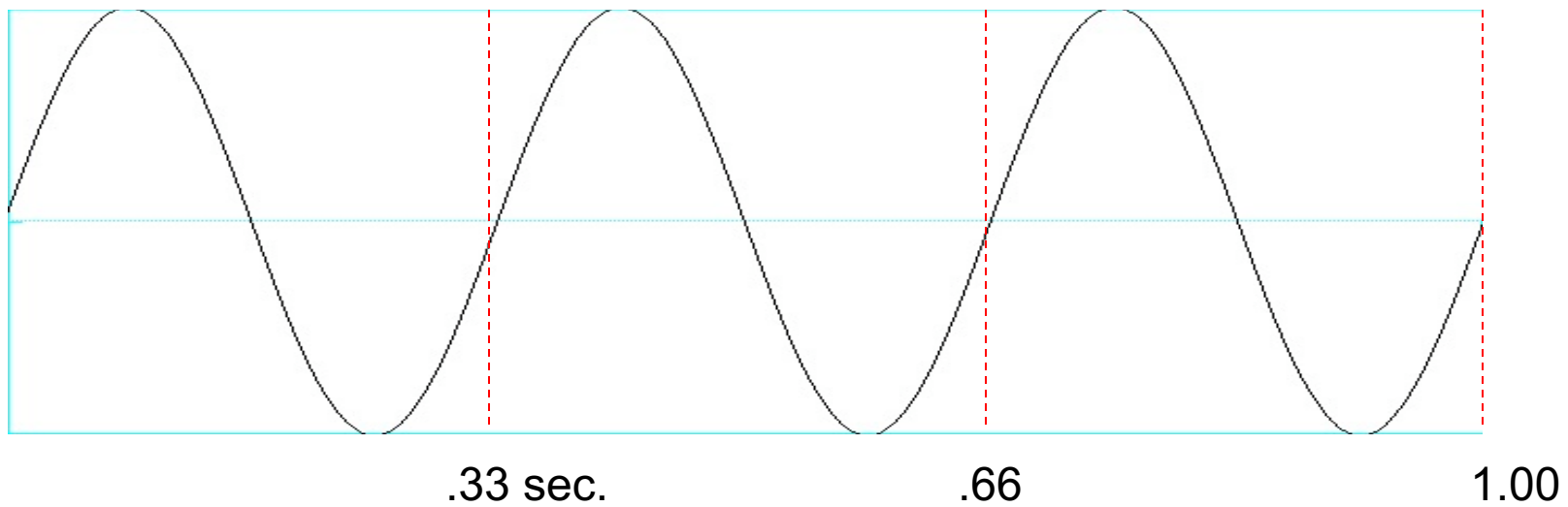
Why?

- Let's look at the flip side of this principle.
 - Think about the *smallest common multiple* of the periods of the component waves.
- Both component waves are **periodic**
 - i.e., they repeat themselves in time.
- The pattern formed by combining these component waves...
 - will only start repeating itself when both waves start repeating themselves **at the same time**.
- Example: 3 Hz sinewave + 5 Hz sinewave

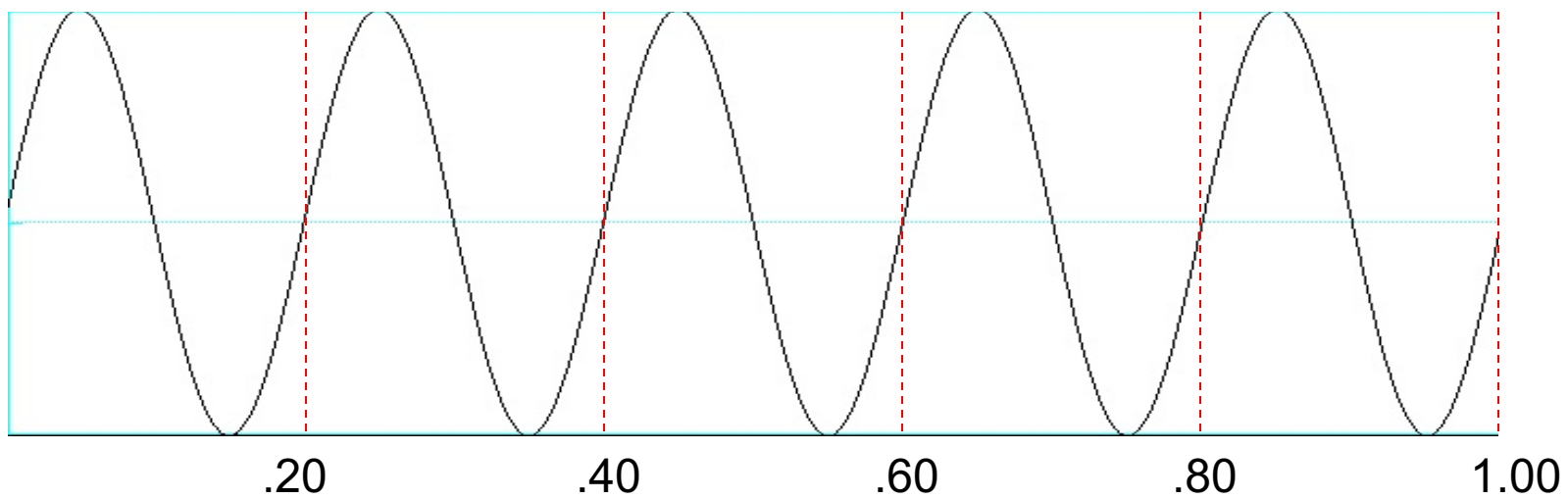
For Example

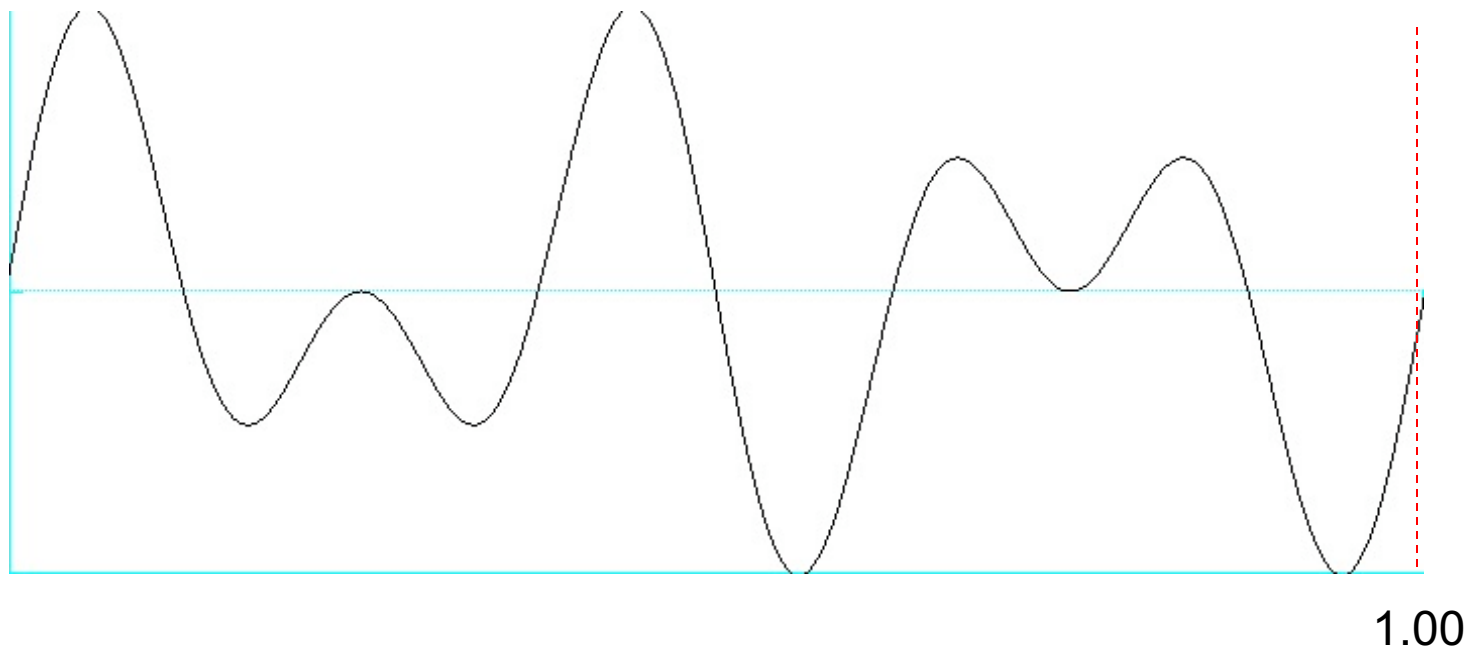
- Starting from 0 seconds:
 - A 3 Hz wave will repeat itself at .33 seconds, .66 seconds, **1 second**, etc.
 - A 5 Hz wave will repeat itself at .2 seconds, .4 seconds, .6 seconds, .8 seconds, **1 second**, etc.
- Again: the pattern formed by combining these component waves...
 - will only start repeating itself when they both start repeating themselves at the same time.
 - i.e., at **1 second**

3 Hz



5 Hz





Combination of 3 and 5 Hz waves

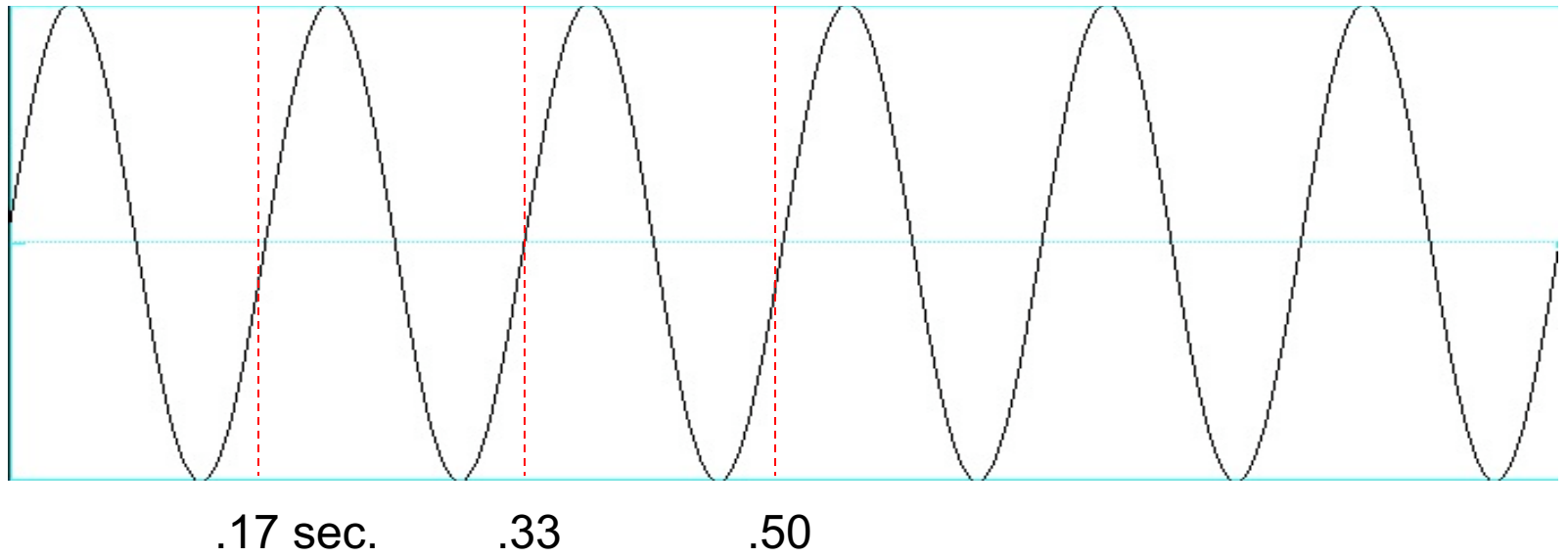
(period = 1 second)

(frequency = 1 Hz)

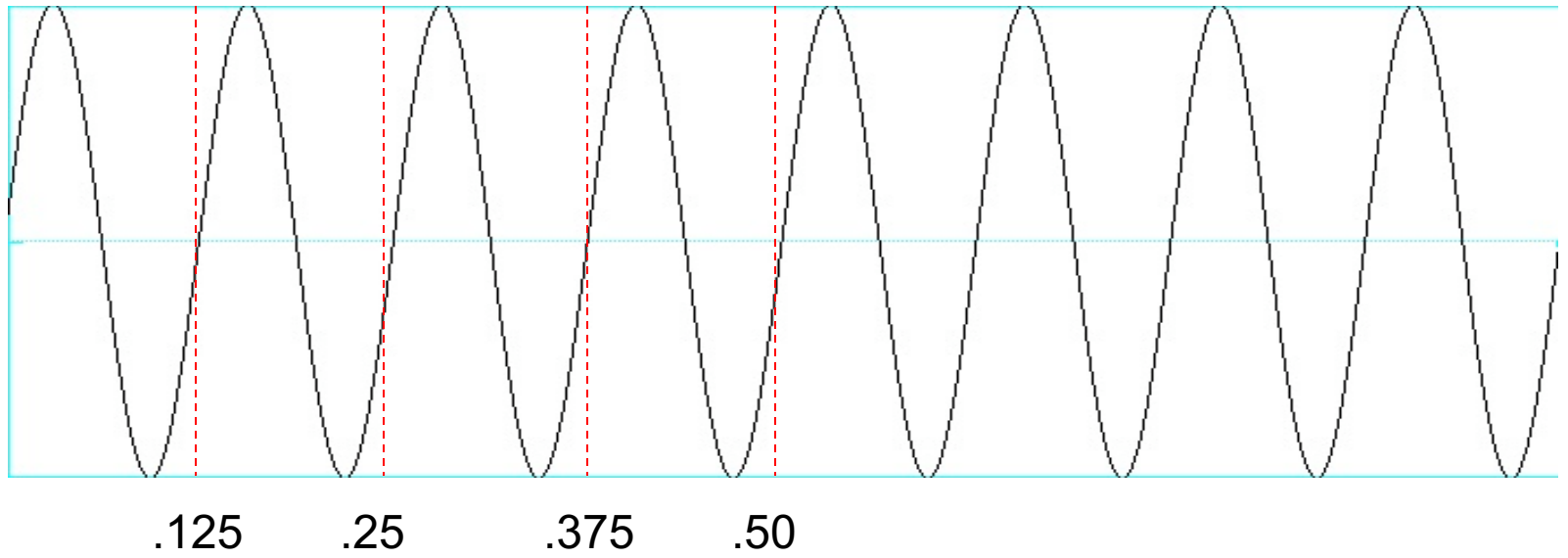
Tidbits

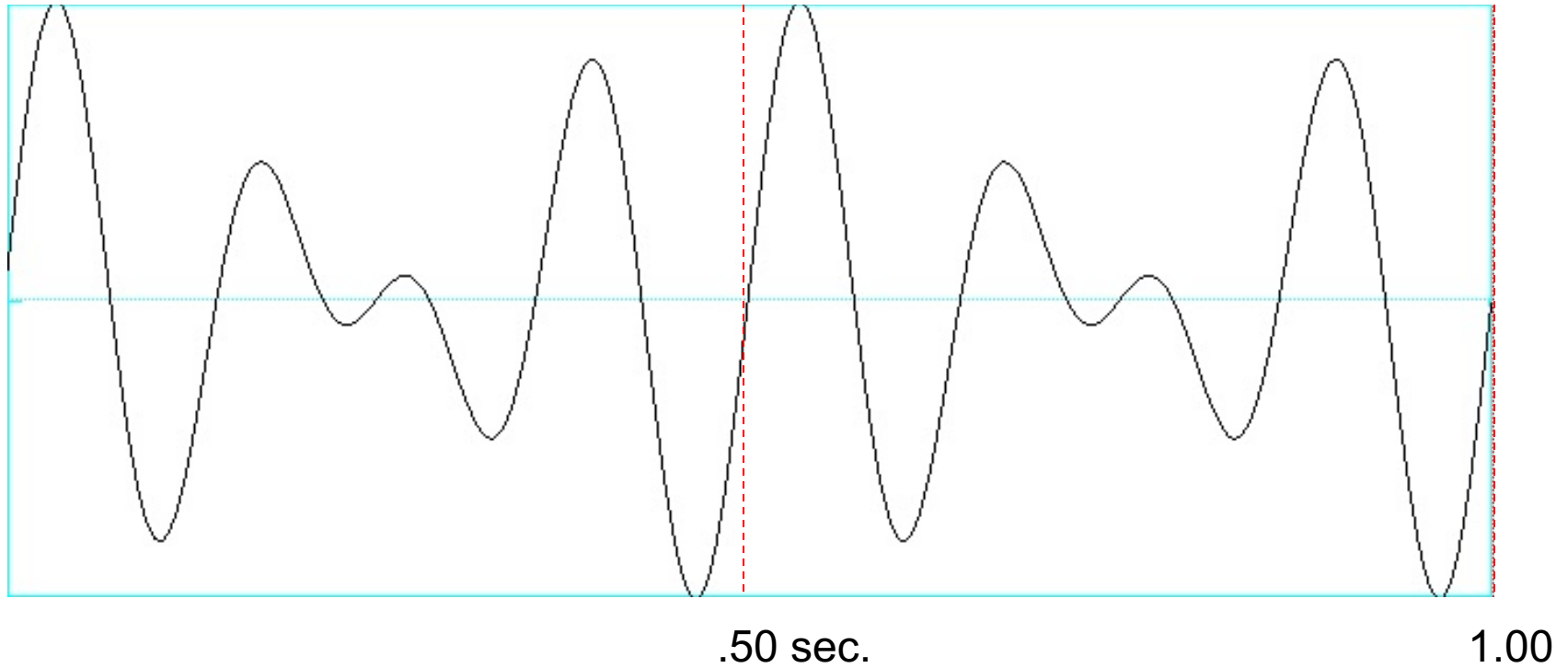
- Important point:
 - Each component wave will complete a whole number of periods within each period of the complex wave.
- Comprehension question:
 - If we combine a 6 Hz wave with an 8 Hz wave...
 - What should the frequency of the resulting complex wave be?
- To ask it another way:
 - What would the period of the complex wave be?

6 Hz



8 Hz





Combination of 6 and 8 Hz waves
(period = .5 seconds)
(frequency = 2 Hz)

Fourier's Theorem

- Joseph Fourier (1768-1830)
 - French mathematician
 - Studied heat and periodic motion
- His idea:
 - **any** complex periodic wave can be constructed out of a combination of different sinewaves.
- The sinusoidal (sinewave) components of a complex periodic wave = **harmonics**



The Dark Side



Fourier's theorem implies:

- sound may be split up into component frequencies...
- just like a prism splits light up into its component frequencies

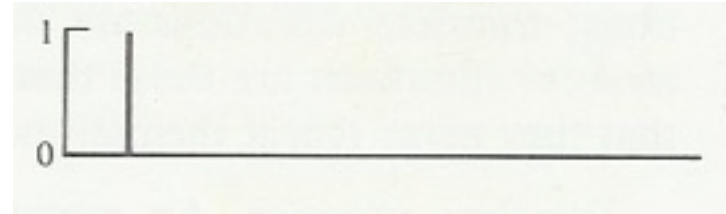
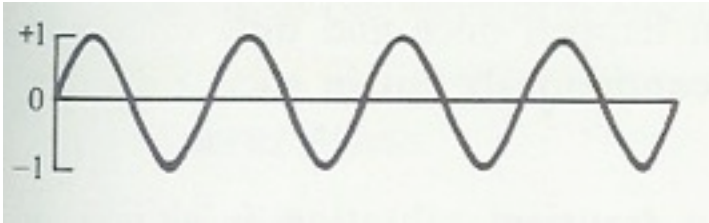
Spectra

- One way to represent complex waves is with waveforms:
 - y-axis: air pressure
 - x-axis: time
- Another way to represent a complex wave is with a **power spectrum** (or **spectrum**, for short).
- Remember, each sinewave has two parameters:
 - amplitude
 - frequency
- A power spectrum shows:
 - intensity (based on amplitude) on the y-axis
 - frequency on the x-axis

Two Perspectives

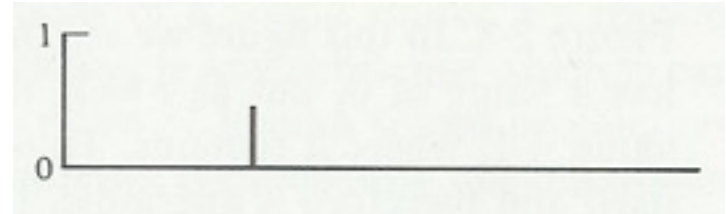
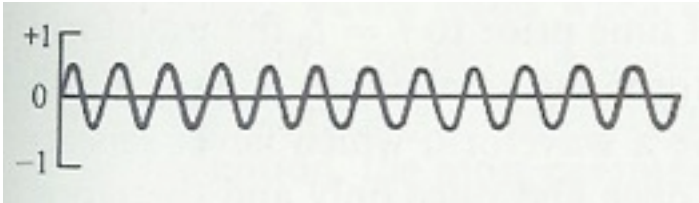
Waveform

Power Spectrum



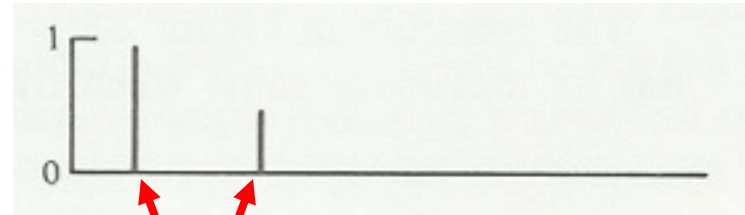
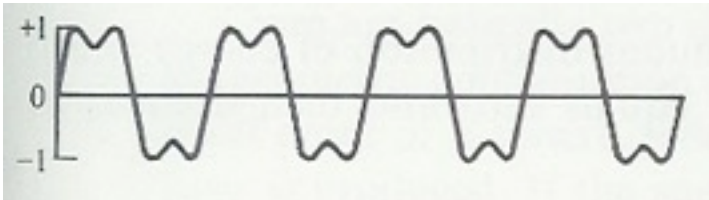
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harmonics

Example

- Go to Praat
- Generate a complex wave with 300 Hz and 500 Hz components.
- Look at waveform and spectral views.
- And so on and so forth.

Fourier's Theorem, part 2

- The component sinusoids (harmonics) of any complex periodic wave:
 - all have a frequency that is an integer multiple of the fundamental frequency of the complex wave.
- This is equivalent to saying:
 - all component waves complete an integer number of periods within each period of the complex wave.

Example

- Take a complex wave with a fundamental frequency of 100 Hz. It can only have harmonics at the following frequencies:
 - Harmonic 1 = 100 Hz
 - Harmonic 2 = 200 Hz
 - Harmonic 3 = 300 Hz
 - Harmonic 4 = 400 Hz
 - Harmonic 5 = 500 Hz
- etc.

Deep Thought Time

- What are the possible harmonic frequencies of a complex wave with a fundamental frequency of 440 Hz?
- Harmonic 1: 440 Hz
- Harmonic 2: 880 Hz
- Harmonic 3: 1320 Hz
- Harmonic 4: 1760 Hz
- Harmonic 5: 2200 Hz
- etc.
- For complex waves, the frequencies of the harmonics will always depend on the fundamental frequency of the wave.

A new spectrum

- Sawtooth wave at 440 Hz
- Also check out a sawtooth wave at 150 Hz

Another Representation

- **Spectrograms**

- = a three-dimensional view of sound
- Incorporate the same dimensions that are found in spectra:
 - Intensity (on the z-axis)
 - Frequency (on the y-axis)
- And add another:
 - Time (on the x-axis)
- Back to Praat, to generate a complex tone with component frequencies of 300 Hz, 2100 Hz, and 3300 Hz

Something Like Speech

- Check this out:




- One of the characteristic features of speech sounds is that they exhibit spectral change over time.

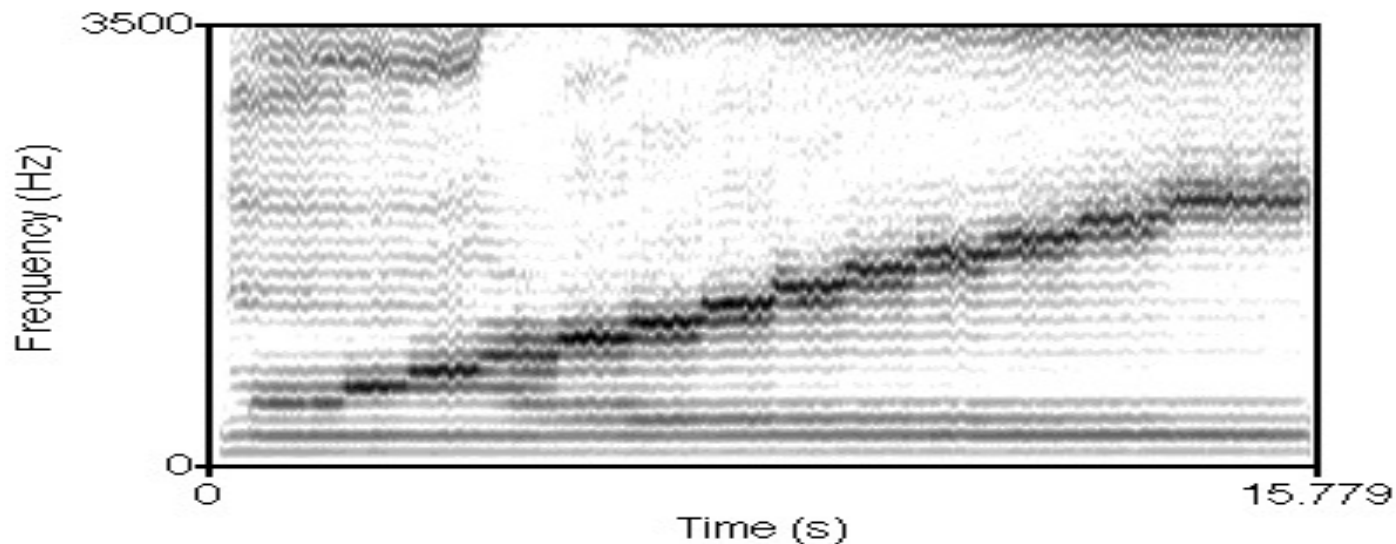
source: <http://www.haskins.yale.edu/featured/sws/swssentences/sentences.html>

Harmonics and Speech

- Remember: trilling of the vocal folds creates a complex wave, with a fundamental frequency.
- This complex wave consists of a series of harmonics.
- The spacing between the harmonics--in frequency--depends on the rate at which the vocal folds are vibrating (F_0).
- We can't change the frequencies of the harmonics independently of each other.
- Q: How do we get the spectral changes we need for speech?

Resonance

- Answer: we change the intensity of the harmonics
- Listen to this: 



source: http://www.let.uu.nl/~audiufon/data/e_boventoon.html