

Logarithms, Decibels, and Intensity: a review sheet

1. Bels

Remember that we defined the **intensity** of a sound, x , with the following equation:

$$I(x) = \log_{10} (x^2 / r^2)$$

where $I(x)$ = the intensity of sound x , as measured in **bels**.

Let's unpack the terms of this equation, to see exactly what they mean.

2. Power

The **power** of a sound is just the square of its amplitude.

Hence, the power of sound x is just x^2 .

3. r^2

The intensity equation includes an r^2 term.

The intensity of a sound, x , is calculated by comparing its power to the power of a reference sound, r . This reference sound is the quietest sound that (healthy, young) human ears can hear.

4. Intensity

The intensity of x is equal to the (base 10) **logarithm** of the ratio of x^2/r^2 .

The concept of **logarithms** is related to the concept of **exponents**.

5. Exponents

Numbers can be multiplied by themselves.

The “square” of a number is the value of that number multiplied by itself.

$$10 * 10 = 100$$

A shorthand for this equation is to put a superscript “2” next to the number that is being multiplied by itself.

$$10^2 = 100$$

The number “2” represents the number of 10s that are being multiplied in the equation.

Similarly, the “cube” of a number, x, is the value of multiplying three of those numbers together.

$$10 * 10 * 10 = 1000$$

$$10^3 = 1000$$

So the ³ here represents that 3 tens have been multiplied in the equation to get 1000.

In general, the number of “x”s being multiplied by each other in an equation is represented by a superscript “y”.

The equation:

$$x^y = z$$

says that z is the value of multiplying y “x”s with each other.

The “y” in this equation is known as the **exponent** of x.

6. Logarithms

Sometimes, we want to know how many times we need to multiply a number by itself to get another number.

For instance, how many “10”s do we need to multiply with each other to get 1000?

3.

To get 10,000?

4.

etc.

A logarithm is a shorthand way of representing that we need to multiply a number, x , with itself z times to get another number, y .

$$\log_x (y) = z$$

Examples:

$$\log_{10} (100) = 2$$

$$\log_{10} (1000) = 3$$

$$\log_{10} (10000) = 4$$

etc.

7. Back to Intensity

Now we can reconsider our definition of **intensity**:

$$I(x) = \log_{10} (x^2 / r^2)$$

The intensity of a sound, x , is the number of times we need to multiply 10 by itself to get the value (x^2 / r^2) .

8. Decibels

A decibel is just a tenth of a bel. So the intensity of a sound x in decibels is:

$$I(x) = 10 * \log_{10} (x^2 / r^2)$$